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Thermal impedances of thin plates

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Abstract

A thin plate of aluminium has been heated by a small power electronic circuit. Using Fourier techniques the thermal impedance of the structure could be measured from the transient temperature behaviour. A simple one-dimensional AC thermal analysis of the aluminium strip is derived and a good comparison between the theory and the experimental results is observed.

Keywords: Thermal impedance; Thin plate; Convective cooling; Heat transfer coefficient; Nyquist plot; Dynamic characterization

1. Introduction

In most books about general heat transfer, a chapter or a section is devoted to the so called thin plate theory [1,2]. The basic assumption is that the temperature in the plate is uniform in the z direction, i.e. perpendicular to the plate. The basic criterion is that the Biot number should be much less then unity or $Bi = ht_s/k \ll 1$. Such condition physically means that a small heat source on the plate gives rise to a heated zone with a diameter much greater than t_s.

In this paper, the thin plate theory will be extended to a time dependent analysis, which will be modelled using phasor notation, a method well known in electric circuit analysis and electromagnetism but almost unused in thermal engineering [3]. If a heat source is connected to the plate, one can define its thermal impedance Z_{th} as the relation between the source temperature increase T_0 and the dissipated power P, both expressed in phasor notation. Like in electric circuit theory, the thermal impedance turns out to be a function of the angular frequency: $Z_{\mathrm{th}}(\mathrm{j}\omega)$. It will be shown in this contribution that by performing measurements at different frequencies, it will be possible to determine the heat transfer coefficient h.

A technique has been developed to measure the thermal impedance from the transient temperature plot versus time. This method was used by several researchers to measure the thermal behaviour of electronic and microelectronic packages [4,5]. It was also found that the particular representation of the thermal impedance in a so called Nyquist plot gives rise to interesting physical interpretations [6,7].

2. Basic theory

Consider a half infinite thin plate, thickness t_s and width b with a heat source of P watt located at $x = 0$ (Fig. 1). Both front and rear side are convectively cooled. If the thickness is sufficiently small, the convective heat release along the small edges at $y = 0$ and $y = b$ can be neglected. If the heat source P is spread over the entire cross section at $x = 0$, one can assume that the temperature will not be dependent upon y or z: $T(x, t)$. The equation for the time dependent problem is then given by

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Nomenclature

a	length of plate (m)	T	temperature distribution (K)
Bi	Biot number (–)	T_{amb}	temperature of ambient air (assumed uniform) (K)
b	width of plate (m)	t	time (s)
C_v	specific volumetric heat of plate (J/m^3 K)	t_s	thickness of plate (m)
f	frequency (Hz)	ω	angular frequency (rad/s)
h	convection coefficient to ambient air (W/m^2 K)	ω_0	characteristic angular frequency (rad/s)
j	imaginary unit $\sqrt{-1}$ (–)	ω_m	angular frequency at minimal Im (Z_{th}) (rad/s)
k	thermal conductivity of plate (W/m K)	$Z_{\mathrm{th}}(\mathrm{j}\omega)$	thermal impedance (K/W)
L	characteristic length (m)		
P	dissipated power (W)		

$$kt_s\frac{\partial^2 T}{\partial x^2} - 2h(T - T_{\mathrm{amb}}) = C_v t_s \frac{\partial T}{\partial t} \tag{1}$$

In (1) both sides are cooled convectively, which explains the factor 2 in the second term.

Without loss of generality one can take from now on $T_{\mathrm{amb}} = 0$ so that T represents the temperature increase above ambient. In phasor notation the Eq. (1) becomes

$$kt_s\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} - 2hT(x) = \mathrm{j}\omega C_v t_s T(x) \tag{2}$$

where the complex function $T(x)$ is the phasor representation of the temperature distribution. Eq. (2) can be rearranged into

$$\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} - \frac{1}{L^2}\left(1 + \frac{\mathrm{j}\omega}{\omega_0}\right)T(x) = 0 \tag{3}$$

where

$$L = \sqrt{\frac{kt_s}{2h}} \tag{4}$$

and

$$\omega_0 = \frac{2h}{C_v t_s}(= 2\pi f_0) \tag{5}$$

Remind that

$$Bi = \frac{1}{2}\left(\frac{t_s}{L}\right)^2 \tag{6}$$

L can be considered as a characteristic length and ω_0 can be interpreted as a characteristic angular frequency. For frequencies much less than f_0, one obtains the DC or the steady state behaviour. For high frequencies above f_0 the temperature behaviour becomes totally different.

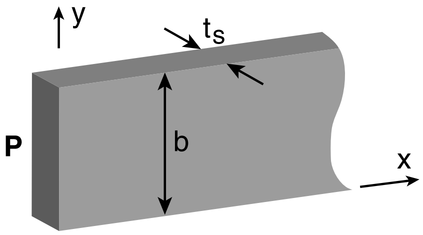

Fig. 1. Thin cooling plate with heat source P.

The boundary condition at $x = 0$ is given by

$$P = -kbt_s\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right)_{x=0} \tag{7}$$

where it has been assumed that no heat is released by convection in the $-x$ direction. If the cooling fin is infinitely long, the solution of (3) is obviously

$$T = T_0 \exp\left(-\sqrt{1 + \mathrm{j}\omega/\omega_0}\,\frac{x}{L}\right) \tag{8}$$

Inserting the solution (8) into the boundary condition (7) gives rise to

$$\begin{aligned} T &= \frac{PL\exp\left(-\sqrt{1+\mathrm{j}\omega/\omega_0}\,\frac{x}{L}\right)}{kt_s b\sqrt{1+\mathrm{j}\omega/\omega_0}} \\ &= \frac{P\exp\left(-\sqrt{1+\mathrm{j}\omega/\omega_0}\,\frac{x}{L}\right)}{b\sqrt{2hkt_s}\sqrt{1+\mathrm{j}\omega/\omega_0}} \end{aligned} \tag{9}$$

Under steady state conditions, (9) is reduced to

$$T = \frac{P}{b\sqrt{2hkt_s}}\exp\left(-\frac{x}{L}\right) \tag{10}$$

which has been verified experimentally on thin ceramic substrates for one and two-dimensional heat flows [8–10].

For an aluminium plate with parameters $C_v = 2.4 \times 10^6$ J/m^3 K, $t_s = 1$ mm and $k = 200$ W/m K and cooled under natural convection with $h = 20$ W/m^2 K one obtains from (4) and (5)

$$L = 7.07\ \mathrm{cm} \quad \text{and} \quad \omega_0 = 0.02\ \mathrm{rad/s} \quad (f_0 = 0.0032\ \mathrm{Hz}) \tag{11}$$

If the length of the cooling fin is a few times the characteristic length L, the plate can be considered as infinitely long, as was assumed in the theoretical analysis.

The thermal impedance of the power source is by definition

$$Z_{\mathrm{th}} = \frac{T_0}{P} = \frac{T(x=0)}{P} = \frac{1}{b\sqrt{2hkt_s}\sqrt{1+\mathrm{j}\omega/\omega_0}} \tag{12}$$

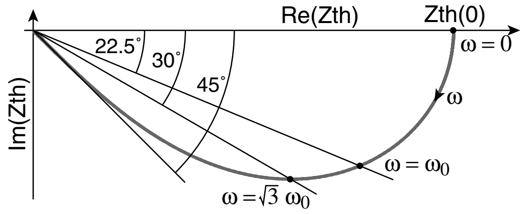

Fig. 2. Graphical representation of the theoretical thermal impedance.

A graphical representation of (12) is shown in Fig. 2. A so called Nyquist plot has been displayed, which shows the imaginary part versus the real part of the thermal impedance using the angular frequency as a parameter. One can prove that the minimum of the curve is obtained for $\omega = \omega_m = \omega_0\sqrt{3}$. The phase is then exactly $\pi/6$. The point corresponding to $\omega = \omega_0$ is found for a phase angle of $\pi/8$. For high frequencies the phase of (12) tends to $\pi/4$.

3. Experimental results

In the experimental measurements, a single chip power amplifier ($10 \times 3.5 \times 10\ \mathrm{mm}^3$) was connected to a thin aluminium plate with a width $b = 6$ cm and a total length a of more than 28 cm. Since the length is much more than the characteristic length (11), the plate can be considered as infinitely long. The thermal parameters for the plate are $k = 200$ W/m K and $C_v = 2.4 \times 10^6\ \mathrm{J/m^3\,K}$. The layout has been drawn in Fig. 3.

The heat source being the silicon chip, its temperature can be easily monitored using the T3ster equipment from MICRED company. The method is based on the fact that all semiconductor characteristics are temperature dependent. During a sufficiently long period, a constant and known power is dissipated in the chip till steady state thermal conditions were reached. At the time $t = 0$, the power was switched off and the junction temperature decay curve $T_j(t)$ was measured. From the T_j versus time t curve, it is quite straightforward to obtain the AC thermal impedance $Z_{th}(j\omega)$ by Fourier techniques.

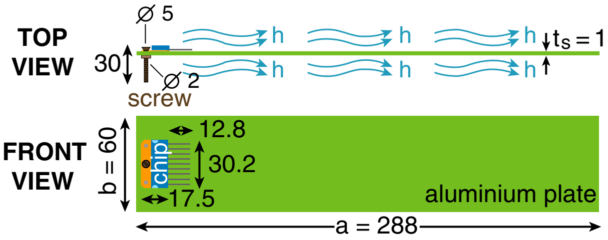

Fig. 3. Layout of the power amplifier and cooling fin (all dimensions in mm).

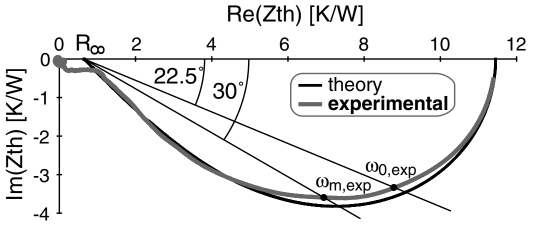

Fig. 4. Experimental Nyquist plot of the thermal impedance.

The experimental results are displayed in Fig. 4. This data has been obtained for a power $P = 5$ W. A closer look at Fig. 4 reveals that the experimental values can be very well fitted to the theoretical plot shown in Fig. 2 provided a small shift to the right is performed. Instead of (12), one has to use

$$Z_{\mathrm{fit}} = R_\infty + \frac{R_0}{\sqrt{1 + j\omega/\omega_0}} \qquad \left(R_0 = \frac{1}{b\sqrt{2hkt_s}}\right) \tag{13}$$

A best fit has been obtained for $R_\infty = 0.645$ K/W. Physically the quantity R_∞ can be easily explained. In Fig. 1 the heat source is in direct contact with the aluminium plate. In our experiment a packed chip screwed to the plate was used. The silicon temperature was recorded in order to obtain the thermal impedance. As a consequence the thermal resistance of the amplifier package should be added to the thermal impedance of the plate as has been done in (13). Due to the small dimensions of the package, its thermal behaviour is limited to the high frequency part of Z_{th} which is the small curve near the origin in Fig. 4. Other measurements and simulations confirm this statement as well [11,12].

From the point corresponding to $Z_{th} = R_\infty$ (Fig. 4) a line has been drawn under an angle of 30°. The intersection point with the experimental curve turns out to correspond to $\omega_{m,\mathrm{exp}} = 0.0377$. Using (5) and $\omega_m = \omega_0\sqrt{3}$ one finds for the heat transfer coefficient

$$h = \frac{\omega_{m,\mathrm{exp}} C_v t_s}{2\sqrt{3}} = 26.3\ \mathrm{W/m^2\,K} \tag{14}$$

At first sight this value is rather high. Using the following classical formula [13,14]:

$$h = 1.485\left(\frac{\Delta T}{b}\right)^{1/4} \tag{15}$$

For $\Delta T = Z_{th}(0)P \approx 50$ K one gets a heat transfer coefficient $h = 7.97\ \mathrm{W/m^2\,K}$, much less than the measured value (14). However, Ellison pointed out that most correlations published in the literature are no longer valid for small dimensions, which are common in electronics [14]. Based on numerous experiments, Ellison presented a small device formula:

$$h = 0.9411\left(\frac{\Delta T}{b}\right)^{0.35} \tag{16}$$

For the same parameter values we now obtain a new value $h = 9.9\ \mathrm{W/m^2\,K}$, higher than the previous one.

In our experiments the amplifier package has a width of 30 mm whereas the cooling plate has $b = 60$ mm. In the theoretical section it was assumed that the heat source is uniformly distributed over the cross section of the cooling plate (Fig. 1). If $b = 30$ mm is introduced in the relations (15) and (16) higher values 9.4 and 12.6 are obtained, but still much lower than the value 26 obtained from our AC analysis. However, other convection heat transfer measurement made on small scaled ceramic substrates yield a heat transfer coefficients up to 25, in good agreement with our present results [15]. It must be emphasised that these measurements were done in steady state conditions using a thermographic camera to detect the substrate temperature.

In steady state conditions $\omega = 0$, the experimental value of the thermal impedance turns out to be $Z_{\mathrm{th}}(0) = 11.4$ K/W. Using (13), we obtain then $R_0 = 11.4 - 0.645 = 10.7$ K/W. According to (13), R_0 is given by

$$R_0 = \frac{1}{b\sqrt{2hkt_s}} \tag{17}$$

or

$$h = \frac{1}{2kt_s b^2 R_0^2} \tag{18}$$

For $b = 60$ mm one obtains $h = 6.06$, whereas for $b = 30$ mm a value $h = 24.24$ is obtained, in good agreement with our AC measurements. One could say that the width of the amplifier package has more influence than the width of the aluminium strip. This can also be understood by the fact that the width of the plate is comparable to the characteristic length (11). Hence the thermal field should rather be considered as two-dimensional.

4. Conclusion

A one-dimensional thermal model has been presented for a thin and long cooling strip. In phasor notation, the calculation could be performed analytically so that a simple formula was found for the thermal impedance. By connecting a small power amplifier to the thin plate, a heat source and at the same time a temperature sensor was available. From the transient thermal behaviour, the thermal impedance could be found by a simple Fourier analysis. A good agreement between the theoretical analysis and the experimental measurements was observed.

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